

# Electroweak Evolution Equations

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## Abstract

Enlarging a previous analysis, where only fermions and transverse gauge bosons were taken into account, we write down infrared-collinear evolution equations for the Standard Model of electroweak interactions computing the full set of splitting functions. Due to the presence of double logs which are characteristic of electroweak interactions (Bloch-Nordsieck violation), new infrared singular splitting functions have to be introduced. We also include corrections related to the third generation Yukawa couplings.

## Introduction

Energy-growing electroweak corrections in the Standard Model have received recently a lot of attention in the literature, being relevant for LHC physics [1], for Next generation of Linear Colliders (NLCs) [2] and for ultrahigh energy cosmic rays [3]. The presence of double logs ( $\log^2 \frac{s}{M^2}$  where  $M$  is the weak scale) in one loop electroweak corrections has been noticed in [4]. One loop effects are typically of the order of 10-20 % at the energy scale of 1 TeV, so that the subject of higher orders and/or resummation of large logarithms has to be addressed. After the observation [5] that double and single logs that appear in the 1 loop expressions are tied to the infrared structure of the theory, all order resummation has been considered at various levels: Leading Log (LL) [6], Next to Leading Log (NLL) [7] and so on. Moreover many fixed order analyses at the one [8] and two loop [9] level have been performed.

Collinear evolution equations are written in general with the purpose of resumming large contributions of logarithmic type by factorizing collinear singularities. This is done by separating a “soft” scale contribution that includes collinear logarithms and a “hard” scale contribution which is free of logarithms and therefore perturbative. However in the electroweak sector at energies  $\sqrt{s}$  much higher than the weak scale  $M \sim M_Z \sim M_W \approx 100$  GeV, logarithms of both collinear and infrared origin appear, even in fully inclusive quantities [10]. In a previous paper [11] we have written infrared evolution equations in the limit of vanishing U(1) coupling and considering only fermions and transverse gauge bosons, showing that the presence of both infrared and collinear singularities can be tackled with by introducing new, infrared singular, splitting functions. The aim of this work is to complete the analysis by writing down infrared evolution equations in the full  $SU(2) \otimes U(1)$  electroweak sector of the Standard Model, including left and right handed fermions, transverse and longitudinal bosons degrees of freedom. Since the third family Yukawa couplings are non negligible, we include their contribution; on the other hand we neglect family mixing effects related to the Cabibbo Kobayashi Maskawa matrix. A complete analysis of collinear singularities in the Standard Model will also have to include QED evolution equations [12] for transverse momenta of the emitted particles lower than the weak scale  $M$ , and QCD DGLAP [13] evolution equations.

The analysis of mass singularities in a spontaneously broken gauge theory like the electroweak sector of the Standard Model has many interesting features. To begin with, initial states like electrons and protons carry nonabelian (isospin) charges; this feature causes the very existence of double logs i.e. the lack of cancellations of virtual corrections with real emission in inclusive observables [10]. Secondly, initial states that are mass eigenstates are not necessarily gauge eigenstates; this causes some interesting mixing phenomena analyzed in [14, 15, 16]. Technically speaking, a complication is due to the fact that the gauge theory Ward Identities are broken by Goldstone boson contributions; however these contributions are proportional to the symmetry breaking scale and are therefore expected to be suppressed to a certain extent in the high energy limit. These contributions have been shown to be negligible at the one

loop level [11]; we assume this to be true at higher orders as well, although we lack a formal proof at the moment. Finally, collinear factorization has been proved at the one loop level [17]; we assume it to be valid to all orders. Ultraviolet logarithms leading to running coupling effects are neglected throughout the paper.

The main results of this paper are contained in eqs. (13-20), that represent the  $SU(2) \otimes U(1)$  electroweak evolution equations for the full SM spectrum. Namely, eqs. (13) are written for matricial structure functions  $\mathcal{F}_{\alpha'\beta'}^{i\alpha\beta}$  in isospin space (see fig. 1). In order to make these equations useful for practical purposes, we have to write them for corresponding scalar quantities. This is done by exploiting the (recovered) isospin and CP symmetry, i.e. by projecting the equations on states of given quantum numbers, diagonalizing, in such a way, the system in block structures characterized by conserved quantum numbers.

The full set of evolution equations are computed working with gauge eigenstates which simplify systematically the evaluation in the high energy regime. We do this by giving in the Appendix a complete classification of possible asymptotic states according to their isospin and CP properties. Then, scalar equations are obtained by a method we introduce here, which consists in performing appropriate traces with respect to isospin leg indices. The final equations for scalar structure functions  $f_{(T,Y;CP)}^i$  are given in (44-60).

Since the overall procedure is quite complicated, in section two we discuss the simple case of left fermions in the initial state and we show how the block diagonalization procedure helps for the numerical and practical evaluation of the evolution equations.

## 2 A working example: left fermions in the $g' \rightarrow 0$ limit

In this section we consider lepton initiated Drell Yan process of type  $e^+(p_1) e(p_2) \rightarrow q(k_1)\bar{q}(k_2) + X^*$  where  $s = 2p_1 \cdot p_2$  is the total invariant mass and  $Q^2 = 2k_1 \cdot k_2$  is the hard scale. We consider double log corrections in relation to the  $SU(2)$  electroweak gauge group, i.e. we work in the limit where the  $U(1)$  coupling  $g'$  is zero. This process has been analyzed in [18, 14], to which we refer for details; we consider it here for convenience and in order to establish our notations. The general formalism used to study electroweak evolution equations for inclusive observables has been set up in [11]; we summarize it here briefly. To begin with, by arguments of unitarity, final state radiation can be neglected when considering inclusive cross sections [18]. Then we are led to consider the dressing of the overlap matrix  $\mathcal{O}_{\alpha\beta}^{\alpha'\beta'} = \langle \beta\beta' | S^+ | \alpha\alpha' \rangle$ ,  $S$  being the  $S$ -matrix, where only initial states indices appear explicitly (see fig. 1).

At the leading level, all order resummation in the soft-collinear region is obtained by a simple expression that involves the t-channel total isospin  $T$  that couples indices  $\alpha, \beta$ :

$$\mathcal{O}^H \rightarrow \mathcal{O}^{resummed} = e^{-\frac{\alpha_W}{4\pi} [T(T+1)] \log^2 \frac{s}{M_w^2}} \mathcal{O}^H \quad (1)$$

$\alpha_W$  being the weak coupling,  $\mathcal{O}_H$  the hard overlap matrix written in terms of the tree level  $S$ -matrix and  $M_w \sim M_z$ .

At subleading order, the dressing by soft and/or collinear radiation is described at all orders by infrared evolution equations, that are  $T$ -diagonal as far as fermions and transverse gauge bosons are concerned [11]. In order to write down the evolution equations for the case of initial left fermions, we first consider one loop corrections. At the one loop level, virtual and real corrections in NLL approximation can be written as:

$$\delta \mathcal{O}_{\alpha\beta} = \frac{\alpha_W}{2\pi} \int_{M^2}^s \frac{dk_{\perp}^2}{k_{\perp}^2} \int_0^1 \frac{dz}{z} \left\{ P_{ff}^R(z) \theta(1-z - \frac{k_{\perp}}{\sqrt{s}}) t_{\beta\beta'}^A t_{\alpha'\alpha}^A \mathcal{O}_{\alpha'\beta'}^H(zp) + \right. \\ \left. P_{gf}^R(z) [t^B t^A]_{\beta\alpha} \mathcal{O}_{AB}^H(zp) + C_f P_{ff}^V(z, \frac{k_{\perp}}{\sqrt{s}}) \mathcal{O}_{\alpha\beta}^H(p) \right\} \quad (2)$$

where  $P_{ff}^R(z)$ ,  $P_{gf}^R(z)$  and  $P_{ff}^V(z, k_{\perp})$  are defined in the Appendix. The indices below the overlap matrix label the kind of particle:  $L$  = Left fermion and  $g$  = gauge boson ; indices  $\alpha, \beta$  refer to the isospin index ( $\alpha = 1$  corresponds to  $\nu$ ,  $\alpha = 2$  to  $e$ ) of the lower legs while upper legs indices are omitted.

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\*note that the process considered, like all others in this paper, is fully inclusive, meaning also  $W, Z$  radiation is included.

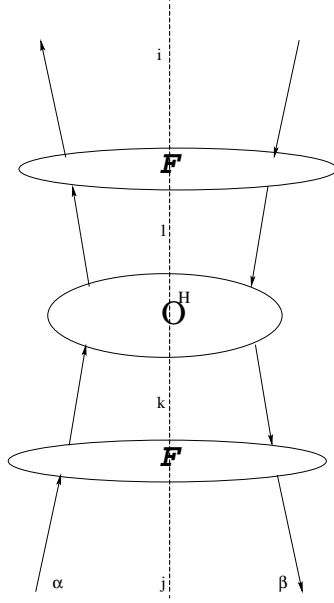


Figure 1: Graphical picture of the factorization formula 3.)

The one loop formula (2) is consistent with a general factorization formula of type (see fig. 1))

$$\mathcal{O}_j^i(p_1, p_2; k_1, k_2) = \int \frac{dz_1}{z_1} \frac{dz_2}{z_2} \sum_{k,l}^{L,R,g} \mathcal{F}_j^k(z_1; s, M^2) \mathcal{O}_k^H(z_1 p_1, z_2 p_2; k_1, k_2) \mathcal{F}_l^i(z_2; s, M^2) \quad (3)$$

where  $i, j$  label the kind of particle ( $L$ =left fermion,  $g$ =gauge boson), and where isospin flavor indices in the overlap function  $\mathcal{O}$  and structure function  $\mathcal{F}$  are understood.

If the factorization formula (3) is assumed to be valid at higher orders as well, the structure functions will satisfy evolution equations with respect to an infrared-collinear cutoff  $\mu$  parameterizing the lowest value of  $k_\perp$ , as follows ( $t = \log \mu^2$ ):

$$-\frac{\partial}{\partial t} \mathcal{F}_j^i{}_{\alpha\beta} = \frac{\alpha_W}{2\pi} \left\{ C_f \mathcal{F}_j^i{}_{\alpha\beta} \otimes P_{ff}^V + [t^C \mathcal{F}_j^i{}^t t^C]_{\alpha\beta} \otimes P_{ff}^R + [t^B t^A]_{\beta\alpha} \mathcal{F}_g^i{}_{AB} \otimes P_{gf}^R \right\} \quad (4)$$

In these equations  $t^A$  denote the isospin matrices in the fundamental representation and  $\mathcal{F}_{\alpha\beta}^i$  denotes the distribution of a particle  $i$  (whose isospin indices are omitted) inside particle  $j$  (with isospin leg indices  $\alpha, \beta$ ).  $\mathcal{F}_j^i{}^t$  is the transpose matrix  $\mathcal{F}_{\beta\alpha}^i$ . Furthermore, we have defined the convolution  $[f \otimes P](x) \equiv \int_x^1 P(z) f(\frac{x}{z}) \frac{dz}{z}$ ; the relevant splitting functions are given in Appendix. Since the index  $i$  is always kept fixed in (4), we will omit it from now on, with the understanding that, for instance,  $\mathcal{F}_j$  collectively denotes all  $\mathcal{F}_j^i$  with any value of  $i$ .

Eqn. (4) is a matricial evolution equation; in order to make it useful we can write the corresponding scalar equations. We do this by exploiting the  $SU(2)_L$  symmetry which allows to classify the states according to their isospin quantum numbers. We couple the lower legs  $\alpha, \beta$  in fig. 1 obtaining the t-channel isospin eigenstates:

$$|\mathbf{T} = 0\rangle = \frac{1}{\sqrt{2}}(|\nu\nu^*\rangle + |ee^*\rangle) \quad |\mathbf{T} = 1\rangle = \frac{1}{\sqrt{2}}(|\nu\nu^*\rangle - |ee^*\rangle) \quad (5)$$

which have  $T_L^3 = 0$  since cross sections always have a given particle on leg  $\alpha$  and its own antiparticle on leg  $\beta$ . We

now project the structure operators  $\mathcal{F}$  on these states, omitting the upper leg indices:

$$f_{(0)} = \frac{\langle \nu\nu^* + ee^* | \mathcal{F} | \rangle_L}{2} = \frac{\mathcal{F}_{\nu\nu} + \mathcal{F}_{ee}}{2} = \frac{1}{2} \text{Tr} [\mathcal{F}] \quad f_{(1)} = \frac{\langle \nu\nu^* - ee^* | \mathcal{F} | \rangle_L}{2} = \frac{\mathcal{F}_{\nu\nu} - \mathcal{F}_{ee}}{2} = \text{Tr} [t^3 \mathcal{F}] \quad (6)$$

Last step in eqs. (6) represents a convenient way to extract the scalar coefficients  $f_j^{(T)}$  from  $\mathcal{F}$ ; namely, by taking appropriate traces with respect to the soft leg  $j$ . For instance  $f_{(0)}$  corresponds to  $\frac{1}{2}(\mathcal{F}_{ee} + \mathcal{F}_{\nu\nu})$  [11] and can be obtained by  $\text{Tr}_j[\mathcal{F}]$ ; here and in the following the trace is taken with respect to the indices of the soft lower scale leg  $j$ . Notice that since gauge and mass eigenstates do not necessarily coincide, we have to introduce “mixed legs” with particles belonging to different gauge representations on leg  $\alpha$  and  $\beta$  (more about this point, in section 4). We label these cases by  $i = LR$  for the mixed left/right fermion leg,  $i = B3$  for the mixed  $W_3 - B$  gauge bosons and  $i = h3$  for the Higgs sector case. These mixing phenomena are interesting by themselves and have been considered in [14, 15, 16, 20] at double log level.

Projecting eq. (4) for instance on the  $T = 0$  component we obtain:

$$-\frac{\partial}{\partial t} \text{Tr}[\mathcal{F}] = \frac{\alpha_W}{2\pi} \left\{ C_f \text{Tr}[\mathcal{F}] \otimes P_{ff}^V + \text{Tr}[t^C \mathcal{F} t^C] \otimes P_{ff}^R + \text{Tr}[t^B t^A] \mathcal{F}_{AB} \otimes P_{gf}^R \right\} \quad (7)$$

where the traces are taken, here and in the following, with respect to the soft leg indices. This gives:

$$-\frac{\partial}{\partial t} f_{(0)} = \frac{\alpha_W}{2\pi} \left( \frac{3}{4} f_{(0)} \otimes (P_{ff}^V + P_{ff}^R) + \frac{3}{4} f_{(0)} \otimes P_{gf}^R \right) \quad (8)$$

after taking into account that  $\text{Tr}[t^B t^A] \mathcal{F}_{AB} = \frac{1}{2} \sum_A \mathcal{F}_{AA} = \frac{1}{2} \text{Tr}[\mathcal{F}]$  and that  $f_{(0)} = \frac{1}{3} \text{Tr}[\mathcal{F}]$ .

After this short introduction about the use of our projection technique, we now briefly show the utility to project on states of definite t-channel quantum number instead to have evolution equations for single particles. Following ref.[11] we can define the projections with definite value of the t-channel isospin  $T = 0, 1, 2$  for fermion and gauge boson

$$f_{(1)} = \text{Tr}\{t^3 \mathcal{F}\} = \frac{f_\nu - f_e}{2}, \quad f_{(0)} = \frac{f_+ + f_3 + f_-}{3}, \quad f_{(1)} = \frac{f_+ - f_-}{2}, \quad f_{(2)} = \frac{f_+ + f_- - 2f_3}{6} \quad (9)$$

where  $f_e = \mathcal{F}_{ee}$ ,  $f_\nu = \mathcal{F}_{\nu\nu}$ ,  $f_+ = \mathcal{F}_{+-}$ ,  $f_- = \mathcal{F}_{-+}$ ,  $f_3 = \mathcal{F}_{33}$ .

We can evaluate the evolution eqs. for the transverse gauge bosons system following the same line of the fermionic ones (see ref.[11] or next chapter). Then, using our projection technique described before we get the scalar equations with definite  $T$  values; precisely there are five equations coupled in three subsets characterized by the  $T = 0, 1, 2$  values:

$$5 \text{ factorized eqs. } \left\{ \begin{array}{l} \left\{ \begin{array}{l} -\frac{\partial}{\partial t} f_{(0)} = \frac{\alpha_W}{2\pi} \left( \frac{3}{4} f_{(0)} \otimes (P_{ff}^R + P_{ff}^V) + \frac{3}{4} f_{(0)} \otimes P_{gf}^R \right) \\ -\frac{\partial}{\partial t} f_{(0)} = \frac{\alpha_W}{2\pi} \left( 2 f_{(0)} \otimes (P_{gg}^R + P_{gg}^V) + \frac{1}{2} (f_{(0)} + f_{(0)}) \otimes P_{fg}^R \right) \end{array} \right. \\ \left\{ \begin{array}{l} -\frac{\partial}{\partial t} f_{(1)} = \frac{\alpha_W}{2\pi} \left( f_{(1)} \otimes P_{ff}^V - \frac{1}{4} f_{(1)} \otimes (P_{ff}^R + P_{ff}^V) + \frac{1}{2} f_{(1)} \otimes P_{gf}^R \right) \\ -\frac{\partial}{\partial t} f_{(1)} = \frac{\alpha_W}{2\pi} \left( f_{(1)} \otimes P_{gg}^V + f_{(1)} \otimes (P_{gg}^R + P_{gg}^V) + \frac{1}{2} (f_{(1)} + f_{(1)}) \otimes P_{fg}^R \right) \end{array} \right. \\ \left\{ -\frac{\partial}{\partial t} f_{(2)} = \frac{\alpha_W}{2\pi} \left( 3 f_{(2)} \otimes P_{gg}^V - f_{(2)} \otimes (P_{gg}^R + P_{gg}^V) \right) \right. \end{array} \right. \quad (10)$$

with similar equations holding for  $f_L^{(T)}$ .

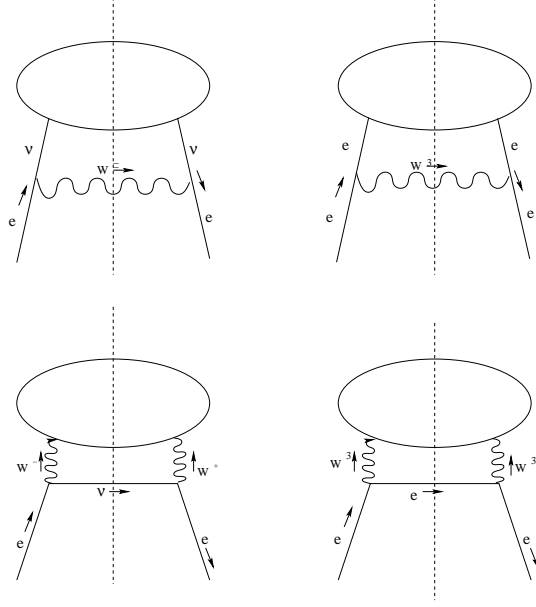


Figure 2: Graphical picture showing the evolution of the single electron component, see eqs. (11).

Eqs.(10) can be converted in evolution equations for the single components (see fig.2):

$$5 \text{ eqs} \left\{ \begin{aligned} -\frac{4\pi}{\alpha_W} \frac{\partial f_\nu}{\partial t} &= f_\nu \otimes (3P_{ff}^V + P_{ff}^R) + 2f_e \otimes P_{ff}^R + 2f_+ \otimes P_{gf}^R + f_3 \otimes P_{gf}^R \\ -\frac{4\pi}{\alpha_W} \frac{\partial f_e}{\partial t} &= f_e \otimes (3P_{ff}^V + P_{ff}^R) + 2f_\nu \otimes P_{ff}^R + 2f_- \otimes P_{gf}^R + f_3 \otimes P_{gf}^R \\ -\frac{2\pi}{\alpha_W} \frac{\partial f_+}{\partial t} &= \frac{f_{\bar{e}} + f_\nu}{2} \otimes P_{fg}^R + f_3 \otimes P_{gg}^R + f_+ \otimes (P_{gg}^R + 2P_{gg}^V) \\ -\frac{2\pi}{\alpha_W} \frac{\partial f_-}{\partial t} &= \frac{f_{\bar{e}} + f_e}{2} \otimes P_{fg}^R + f_3 \otimes P_{gg}^R + f_- \otimes (P_{gg}^R + 2P_{gg}^V) \\ -\frac{2\pi}{\alpha_W} \frac{\partial f_3}{\partial t} &= \frac{f_{\bar{e}} + f_e + f_{\bar{\nu}} + f_\nu}{4} \otimes P_{fg}^R + (f_- + f_+) \otimes P_{gg}^R + 2f_3 \otimes P_{gg}^V \end{aligned} \right. \quad (11)$$

Even if two systems (10,11) are equivalent, eqs. (11) are true  $5 \times 5$  system of differential equations, while the reduction to the block diagonal form of (10) generates a set of  $2 \times 2$ ,  $2 \times 2$  and  $1 \times 1$  coupled equations.

To appreciate the block diagonalization, let us take the case of an initial fermionic parton (an electron). In this case we have only the  $T = 0$  and  $T = 1$  components (the fermionic system cannot couple with the  $T = 2$  projection) so, while the use of eqs. (10) requires the solution of only the  $2 \times 2$  plus  $2 \times 2$  system, the use of eqs. (11) always forces to solve the full  $5 \times 5$  set of eqs. This simplification is particularly important when the full particle spectrum of the SM is taken into account (see next chapter).

Finally, the last step in obtaining the all order resummed overlap matrix, requires the evolution of the  $f_{(T)}^i$ 's according to eqn. (10) with appropriate initial conditions, and inserting the evolved  $f_{(T)}^i$ 's into (3). This can be done by exploiting the recovered isospin symmetry, which allows us to write:

$$\hat{\mathcal{O}}_j^i(p_1, p_2; k_1, k_2) = \sum_T \int \frac{dz_1}{z_1} \frac{dz_2}{z_2} \sum_{k,l}^{L,R,g} \hat{f}_j^k(z_1; s, M^2) \hat{\mathcal{O}}_k^l{}_{(T)}^H(z_1 p_1, z_2 p_2; k_1, k_2) \hat{f}_l^i(z_2; s, M^2) \quad (12)$$

### 3 Full EW evolution equations

Proceeding in analogy with previous section, we now introduce longitudinal gauge bosons and consider the full  $SU(2) \otimes U(1)$  electroweak group. According to the equivalence theorem, we replace longitudinal gauge bosons with

the corresponding Goldstone bosons. We choose to work in an axial gauge so that this substitution can be done without higher order corrections in the definition of the asymptotic states [21].

Our notation goes as follows:

- $\mathcal{F}_{\alpha\beta}^{L_i}$  represent structure functions for left fermions of the  $i$ -family, including leptons and quarks<sup>†</sup>; indices  $\alpha, \beta = 1, 2$  correspond to  $\nu, e$  or  $u, d$  for the first family ( $i = 1$ ) and so on.  $\mathcal{F}_{\alpha\beta}^{L_i}$  indicates the structure function for the corresponding antifermions.
- $\mathcal{F}_{R_i}$  stand for right fermions in the  $i$ th family. We work in the SM and do not consider right neutrinos.  $\mathcal{F}_{R_i}$  is for right antifermions.
- $\mathcal{F}_{LR_i}$  represent the “mixed legs” case where the left leg is for a left fermion and the right leg for a right fermion of the same charge for the  $i$ -family, or viceversa. Such structure functions are relevant only for the case of initial transversely polarized beams [20].
- $\mathcal{F}_{\phi}^{ab}$  represent structure functions for the Goldstone ( $\phi_1, \phi_2, \phi_3$ ) - Higgs ( $h$ ) sector. The Goldstone modes are related to the corresponding longitudinal gauge bosons. Here  $a, b = 1, 2, 3, 4$  stand for  $(\phi_1, \phi_2, \phi_3, h)$ .
- $\mathcal{F}_g^{AB}$  stand for transverse  $W_A$  gauge bosons belonging to the SU(2) sector:  $A, B = 1, 2, 3$ .
- $\mathcal{F}_{\mathcal{B}}$  is the structure function for the U(1)  $\mathcal{B}$  gauge boson.
- $\mathcal{F}_{\mathcal{B}3}$  is the “mixed leg” case involving  $\mathcal{B}, W_3$  transverse gauge bosons.

We choose to work in gauge eigenstate basis in order to represent the various structure functions. This basis is convenient for calculations since at very high energy we can consider massless propagators and therefore avoid the complications arising from mass insertions. Notice however that the asymptotic states appearing in the  $S$  matrix are truly mass eigenstates, so we need to rotate to the physical base as a final step.

The leading one loop graphs contributing, in the axial gauge, to the splitting functions are shown in fig.(3). The kinematical structure is the same as in QCD; however the group structure is much more complicated due to the absence of isospin averaging on the initial states, in contrast with the corresponding (unbroken) QCD case where physical quantities are averaged over color. This implies uncanceled infrared singularities and the introduction of a new kind of infrared singular splitting functions [14].

We define  $\alpha_W \equiv \frac{g^2}{4\pi}$ ,  $\alpha_Y \equiv \frac{g'^2}{4\pi}$ ; the various matrices  $t^A, \mathcal{T}^c, \dots$  appearing in eqs. (13) are defined in the appendix.  $Y$  is the hypercharge operator, which is a diagonal matrix with appropriate eigenvalues in the different representations (left/right fermions and antifermions, longitudinal gauge bosons and so on). We now write down the infrared evolution equations for the structure functions in matrix form. Family indices here are always understood; note that the Yukawa coupling contributions proportional to the matrix  $\mathcal{H}$  defined in the appendix are present only for the third family.

$$\begin{aligned}
-\frac{\partial}{\partial t} \mathcal{F}_{\alpha\beta}^L &= \frac{\alpha_W}{2\pi} \left\{ C_f \mathcal{F}_{\alpha\beta}^L \otimes P_{ff}^V + (t^C \mathcal{F} t^C)_{\beta\alpha} \otimes P_{ff}^R + (t^B t^A)_{\beta\alpha} \mathcal{F}_{AB} \otimes P_{gf}^R \right\} + \\
&\quad \frac{\alpha_Y}{2\pi} \left\{ \frac{1}{2} (Y^2 \mathcal{F} + \mathcal{F} Y^2)_{\alpha\beta} \otimes P_{ff}^V + (Y \mathcal{F} Y)_{\alpha\beta} \otimes P_{ff}^R + (Y^2)_{\alpha\beta} \mathcal{F}_{\mathcal{B}} \otimes P_{gf}^R \right\} + \frac{\sqrt{\alpha_W \alpha_Y}}{2\pi} \left\{ (t^3 Y + Y t^3)_{\alpha\beta} \mathcal{F}_{\mathcal{B}3} \otimes P_{gf}^R \right\} + \\
&\quad \frac{1}{32\pi^2} \left\{ \sum_a (\Psi^a \cdot \mathcal{H} \cdot \mathcal{H}^+ \cdot \Psi^{a+})_{\alpha\alpha} \mathcal{F}_{\alpha\beta}^L \otimes \mathcal{P}_{LL}^V + (\Psi^a \cdot \mathcal{H} \mathcal{F}_R^t \mathcal{H}^+ \cdot \Psi^{a+})_{\beta\alpha} \otimes P_{RL}^R + (\Psi^b \cdot \mathcal{H} \cdot \mathcal{H}^+ \cdot \Psi^{a+})_{\beta\alpha} \mathcal{F}_{\phi}^{ab} \otimes P_{\phi L}^R \right\}; \\
-\frac{\partial}{\partial t} \mathcal{F}_{\alpha\beta}^R &= \frac{\alpha_W}{2\pi} \left\{ C_f \mathcal{F}_{\alpha\beta}^R \otimes P_{ff}^V + (t^C \mathcal{F} t^C)_{\alpha\beta} \otimes P_{ff}^R + \mathcal{F}_{AB} (t^A t^B)_{\alpha\beta} \otimes P_{gf}^R \right\} + \\
&\quad \frac{\alpha_Y}{2\pi} \left\{ \frac{1}{2} (Y^2 \mathcal{F} + \mathcal{F} Y^2)_{\alpha\beta} \otimes P_{ff}^V + (Y \mathcal{F} Y)_{\alpha\beta} \otimes P_{ff}^R + (Y^2)_{\alpha\beta} \mathcal{F}_{\mathcal{B}} \otimes P_{gf}^R \right\} + \frac{\sqrt{\alpha_W \alpha_Y}}{2\pi} \left\{ (t^3 Y + Y t^3)_{\alpha\beta} \mathcal{F}_{\mathcal{B}3} \otimes P_{gf}^R \right\} +
\end{aligned} \tag{13}$$

<sup>†</sup>Quarks structure functions (valid for  $L, R$  or  $LR$  type) are averaged over initial color  $\mathcal{F} \equiv \frac{1}{N_c} \sum_{color_{quarks}} \mathcal{F}$  with  $N_c = 3$ .

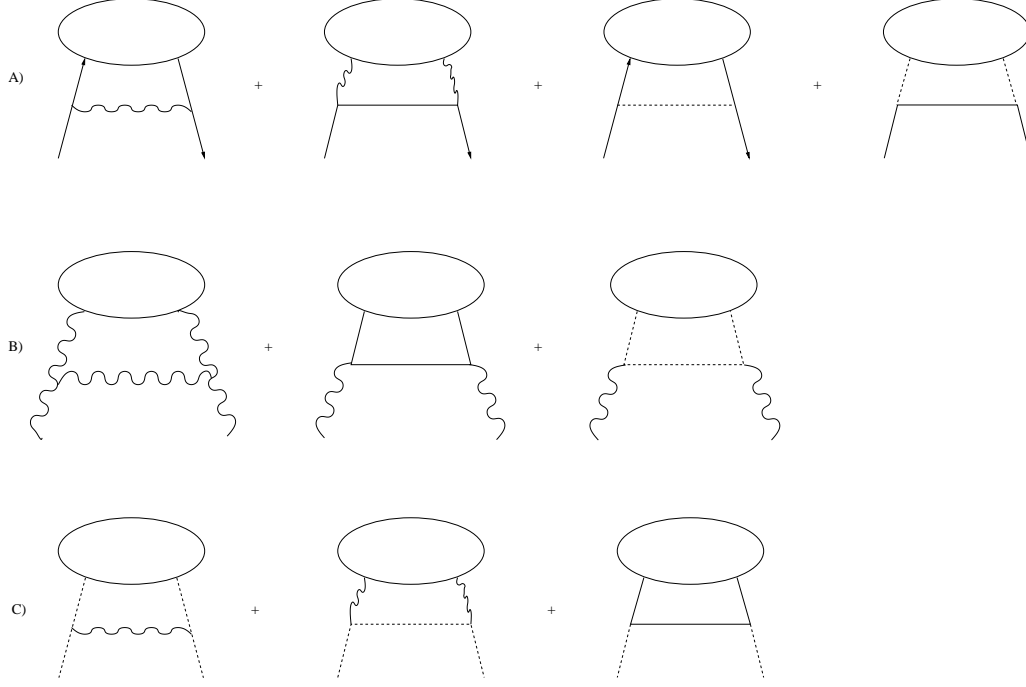


Figure 3: Leading real emission Feynman diagrams in axial gauge: A) Feynman diagrams contributing to the evolution of the fermionic structure functions; B) Feynman diagrams contributing to the evolution of the transverse gauge boson structure functions; C) Feynman diagrams contributing to the evolution of the scalar structure functions. The wavy lines are transverse gauge bosons, dashed lines stay for Higgs sector particles and straight lines for fermions.

$$\begin{aligned}
& \frac{1}{32\pi^2} \left\{ \sum_a (\Psi^a \cdot \mathcal{H} \cdot \mathcal{H}^+ \cdot \Psi^{a+})_{\alpha\alpha} \mathcal{F}_{\alpha\beta} \otimes \mathcal{P}_{LL}^V + (\Psi^a \cdot \mathcal{H} \mathcal{F}_R \mathcal{H}^+ \cdot \Psi^{a+})_{\alpha\beta} \otimes P_{RL}^R + (\Psi^a \cdot \mathcal{H} \cdot \mathcal{H}^+ \cdot \Psi^{b+})_{\alpha\beta} \mathcal{F}_{\phi ab} \otimes P_{\phi L}^R \right\}; \\
-\frac{\partial}{\partial t} \mathcal{F}_{\alpha\beta} &= \frac{\alpha_Y}{2\pi} y_R^2 \left\{ \mathcal{F}_R \otimes P_{ff}^V + \mathcal{F}_R \otimes P_{ff}^R + \mathcal{F}_B \otimes P_{gf}^R \right\} + \\
& \frac{1}{32\pi^2} \left\{ \sum_a (\mathcal{H}^+ \cdot \Psi^{a+} \cdot \Psi^a \cdot \mathcal{H})_{\alpha\alpha} \mathcal{F}_{\alpha\beta} \otimes \mathcal{P}_{RR}^V + (\mathcal{H}^+ \cdot \Psi^{a+} \mathcal{F}_L^t \Psi^a \cdot \mathcal{H})_{\beta\alpha} \otimes P_{LR}^R + (\mathcal{H}^+ \cdot \Psi^{b+} \cdot \Psi^a \cdot \mathcal{H})_{\beta\alpha} \mathcal{F}_{\phi ab} \otimes P_{\phi R}^R \right\}; \\
-\frac{\partial}{\partial t} \mathcal{F}_{\alpha\beta} &= \frac{\alpha_Y}{2\pi} y_R^2 \left\{ \mathcal{F}_R \otimes P_{ff}^V + \mathcal{F}_R \otimes P_{ff}^R + \mathcal{F}_B \otimes P_{gf}^R \right\} + \\
& \frac{1}{32\pi^2} \left\{ \sum_a (\mathcal{H}^+ \cdot \Psi^{a+} \cdot \Psi^a \cdot \mathcal{H})_{\alpha\alpha} \mathcal{F}_{\alpha\beta} \otimes \mathcal{P}_{RR}^V + (\mathcal{H}^+ \cdot \Psi^{a+} \mathcal{F}_L^t \Psi^a \cdot \mathcal{H})_{\alpha\beta} \otimes P_{LR}^R + (\mathcal{H}^+ \cdot \Psi^{a+} \cdot \Psi^b \cdot \mathcal{H})_{\alpha\beta} \mathcal{F}_{\phi ab} \otimes P_{\phi R}^R \right\}; \\
-\frac{\partial}{\partial t} \mathcal{F}_{ab} &= \frac{\alpha_W}{2\pi} \left\{ C_f \mathcal{F}_{\phi ab} \otimes P_{\phi\phi}^V + (T_L^C \mathcal{F}_\phi T_L^C)_{ab} \otimes P_{\phi\phi}^R + \mathcal{F}_{AB} (T_L^B T_L^A)_{ba} \otimes P_{g\phi}^R \right\} + \\
& \frac{\alpha_Y}{2\pi} \left\{ \frac{1}{2} (Y_\phi^2 \mathcal{F} + \mathcal{F} Y_\phi^2)_{ab} \otimes P_{\phi\phi}^V + (Y \mathcal{F} Y)_{ab} \otimes P_{\phi\phi}^R + (Y^2)_{ab} \mathcal{F}_B \otimes P_{g\phi}^R \right\} + \frac{\sqrt{\alpha_W \alpha_Y}}{2\pi} \left\{ (T_L^3 Y + Y T_L^3)_{ab} \mathcal{F}_B \otimes P_{g\phi}^R \right\} + \\
& \frac{1}{32\pi^2} \left\{ Tr(\Psi^b \cdot \mathcal{H} \cdot \mathcal{H}^+ \cdot \Psi^{a+} + \Psi^a \cdot \mathcal{H} \cdot \mathcal{H}^+ \cdot \Psi^{b+}) \mathcal{F}_{\phi ab} \otimes \mathcal{P}_{\phi\phi}^V + \right. \\
& Tr[(\Psi^a \cdot \mathcal{H} \cdot \mathcal{H}^+ \cdot \Psi^{b+}) \mathcal{F}_L^t] \otimes P_{L\phi}^R + Tr[(\Psi^b \cdot \mathcal{H} \cdot \mathcal{H}^+ \cdot \Psi^{a+}) \mathcal{F}_L] \otimes P_{L\phi}^R + \\
& Tr[(\mathcal{H}^+ \cdot \Psi^{a+} \cdot \Psi^b \cdot \mathcal{H}) \mathcal{F}_R^t] \otimes P_{R\phi}^R + Tr[(\mathcal{H}^+ \cdot \Psi^{b+} \cdot \Psi^a \cdot \mathcal{H}) \mathcal{F}_R] \otimes P_{R\phi}^R \Big\}; \\
-\frac{\partial}{\partial t} \mathcal{F}_{AB} &= \frac{\alpha_W}{2\pi} \left\{ C_g \mathcal{F}_{AB} \otimes P_{gg}^V + (T_V^C \mathcal{F}_g T_V^C)_{AB} \otimes P_{gg}^R + \left( \sum_L Tr[t_L^B \mathcal{F}_L^t t_L^A] + \sum_{\bar{L}} Tr[t_{\bar{L}}^A \mathcal{F}_{\bar{L}}^t t_{\bar{L}}^B] \right) \otimes P_{fg}^R + Tr[T_L^B \mathcal{F}_\phi^t T_L^A] \otimes P_{\phi g}^R \right\};
\end{aligned}$$

$$\begin{aligned}
-\frac{\partial}{\partial t} \mathcal{F} &= \frac{\alpha_Y}{2\pi} \left\{ \mathcal{F}_{\mathcal{B}} \otimes P_{\mathcal{B}\mathcal{B}}^R + \sum_{F=L, \bar{L}, R, \bar{R}} \text{Tr} \left[ Y_{\mathcal{F}} \mathcal{F} Y \right] \otimes P_{fg}^R + \text{Tr} \left[ Y_{\mathcal{F}} \mathcal{F} Y \right] \otimes P_{\phi g}^R \right\}; \\
-\frac{\partial}{\partial t} \mathcal{F} &= \frac{\alpha_W}{2\pi} \left\{ \frac{C_g}{2} \mathcal{F}_{\mathcal{B}3} \otimes P_{gg}^V \right\} + \frac{\sqrt{\alpha_W \alpha_Y}}{2\pi} \left\{ \sum_{F=L, \bar{L}} \text{Tr} \left[ Y_{\mathcal{F}} \mathcal{F} t^3 \right] \otimes P_{fg}^R + \text{Tr} \left[ Y_{\mathcal{F}} \mathcal{F} T_L^3 \right] \otimes P_{\phi g}^R \right\} + \frac{\alpha_Y}{2\pi} \left\{ \frac{1}{2} \mathcal{F}_{\mathcal{B}3} \otimes P_{\mathcal{B}\mathcal{B}}^V \right\}; \\
-\frac{\partial}{\partial t} \mathcal{F}_{\alpha\beta} &= \frac{\alpha_W}{2\pi} \left\{ \frac{C_f}{2} \mathcal{F}_{\alpha\beta} \otimes P_{ff}^V \right\} + \frac{\alpha_Y}{2\pi} \left\{ \frac{1}{2} (y_L^2 + y_R^2) \mathcal{F}_{\alpha\beta} \otimes P_{ff}^V + y_L y_R \mathcal{F}_{\alpha\beta} \otimes P_{ff}^R \right\} + \frac{1}{32\pi^2} \left\{ \frac{1}{2} \sum_a (\Psi^a \cdot \mathcal{H} \cdot \mathcal{H}^+ \cdot \Psi^{a+})_{\alpha\alpha} \right. \\
&\quad \left. \mathcal{F}_{\alpha\beta} \otimes P_{LL}^V + \frac{1}{2} \sum_a (\mathcal{H}^+ \cdot \Psi^{a+} \cdot \Psi^a \cdot \mathcal{H})_{\alpha\alpha} \mathcal{F}_{\alpha\beta} \otimes P_{RR}^V + \sum_a (\mathcal{H}^+ \cdot \Psi^{a+} \cdot \mathcal{F}_{LR}^t \cdot \mathcal{H}^+ \cdot \Psi^{a+})_{\beta\alpha} \otimes P_{LR}^R \right\}; \\
-\frac{\partial}{\partial t} \mathcal{F}_{\alpha\beta} &= \frac{\alpha_W}{2\pi} \left\{ \frac{C_f}{2} \mathcal{F}_{\alpha\beta} \otimes P_{ff}^V \right\} + \frac{\alpha_Y}{2\pi} \left\{ \frac{1}{2} (y_L^2 + y_R^2) \mathcal{F}_{\alpha\beta} \otimes P_{ff}^V + y_L y_R \mathcal{F}_{\alpha\beta} \otimes P_{ff}^R \right\} + \frac{1}{32\pi^2} \left\{ \frac{1}{2} \sum_a (\Psi^a \cdot \mathcal{H} \cdot \mathcal{H}^+ \cdot \Psi^{a+})_{\alpha\alpha} \right. \\
&\quad \left. \mathcal{F}_{\alpha\beta} \otimes P_{LL}^V + \frac{1}{2} \sum_a (\mathcal{H}^+ \cdot \Psi^{a+} \cdot \Psi^a \cdot \mathcal{H})_{\alpha\alpha} \mathcal{F}_{\alpha\beta} \otimes P_{RR}^V + \sum_a (\Psi^a \cdot \mathcal{H} \cdot \mathcal{F}_{\bar{L}\bar{R}} \cdot \Psi^a \cdot \mathcal{H})_{\alpha\beta} \otimes P_{LR}^R \right\};
\end{aligned}$$

We now proceed and reduce eqs. (13) to scalar equations. We work in the limit of light Higgs; the relevant symmetry is then the (recovered)  $\text{SU}(2) \otimes \text{U}(1)$  gauge group, and the states can be classified according to the total t-channel isospin  $\mathbf{T}$  and the total t-channel hypercharge  $\mathbf{Y}$ ; however we have to add the conserved quantum number  $CP$  in order to provide a complete classification of the states (see Appendix). Then, our structure functions are labeled by  $(\mathbf{T}, \mathbf{Y}, CP)$ . Furthermore, we have to consider the “mixed cases”: left fermion-right fermion when we are in presence of transverse polarized initial beams [20],  $B - W_3$  mixing when the asymptotic states are  $\gamma$ 's or transverse  $Z$  [14] and  $h - \phi_3$  for longitudinal Goldstone modes [15]. While we refer to the cited works for details, here we stress the fact that we work in the  $(\phi_1, \phi_2, \phi_3, h)$  rather than the  $1, 2, \bar{1}, \bar{2}$  basis used in [15]. The sum  $\sum_i N_c^i$  is over all the fermions where  $N_c^i = 1$  for leptons and  $N_c^i = 3$  for quarks.

The scalar evolution equations, using definitions (44-60) and eqs. (13) can finally be written as:

$$\left\{ \begin{aligned}
& -\frac{\partial}{\partial t} f_{R_i}(0,0,+) = \frac{\alpha_Y}{2\pi} \left( y_{R_i}^2 f_{R_i}(0,0,+) \otimes (P_{ff}^V + P_{ff}^R) + y_{R_i}^2 f_{\mathcal{B}}(0,0,+) \otimes P_{gf}^R \right) + \\
& \quad \frac{\delta_{i3}}{32\pi^2} h_R^2 \left( 4 f_{R_3}(0,0,+) \otimes P_{RR}^V + 4 f_{L_3}(0,0,+) \otimes P_{LR}^R + 4 f_{\phi}(0,0,+) \otimes P_{\phi R}^R \right); \\
& -\frac{\partial}{\partial t} f_{\mathcal{B}}(0,0,+) = \frac{\alpha_Y}{2\pi} \left( f_{\mathcal{B}}(0,0,+) \otimes P_{\mathcal{B}\mathcal{B}}^V + 2 \sum_i N_c^i \left( 2 y_{L_i}^2 f_{L_i}(0,0,+) + \sum_R y_{R_i}^2 f_{R_i}(0,0,+) \right) \otimes P_{fg}^R + f_{\mathcal{B}}(0,0,+) \otimes P_{\phi g}^R \right); \\
& -\frac{\partial}{\partial t} f_{L_i}(0,0,+) = \frac{\alpha_W}{2\pi} \left( \frac{3}{4} f_{L_i}(0,0,+) \otimes (P_{ff}^V + P_{ff}^R) + \frac{3}{4} f_{g}(0,0,+) \otimes P_{gf}^R \right) + \frac{\alpha_Y}{2\pi} \left( y_{L_i}^2 f_{L_i}(0,0,+) \otimes (P_{ff}^V + P_{ff}^R) + y_{L_i}^2 f_{\mathcal{B}}(0,0,+) \otimes P_{gf}^R \right) + \\
& \quad \frac{\delta_{i3}}{32\pi^2} \left( 2 (h_t^2 + h_b^2) f_{L_3}(0,0,+) \otimes P_{LL}^V + 2 \sum_R h_{R_3}^2 f_{R_3}(0,0,+) \otimes P_{RL}^R + 2 (h_t^2 + h_b^2) f_{\phi}(0,0,+) \otimes P_{\phi L}^R \right); \\
& -\frac{\partial}{\partial t} f_g(0,0,+) = \frac{\alpha_W}{2\pi} \left( 2 f_g(0,0,+) \otimes (P_{gg}^V + P_{gg}^R) + \sum_i N_c^i f_{L_i}(0,0,+) \otimes P_{fg}^R + f_{\phi}(0,0,+) \otimes P_{\phi g}^R \right); \\
& -\frac{\partial}{\partial t} f_{\phi}(0,0,+) = \frac{\alpha_W}{2\pi} \left( \frac{3}{4} f_{\phi}(0,0,+) \otimes (P_{\phi\phi}^V + P_{\phi\phi}^R) + \frac{3}{4} f_g(0,0,+) \otimes P_{g\phi}^R \right) + \frac{\alpha_Y}{2\pi} \left( \frac{1}{4} f_{\phi}(0,0,+) \otimes (P_{\phi\phi}^R + P_{\phi\phi}^V) + \frac{1}{4} f_{\mathcal{B}}(0,0,+) \otimes P_{g\phi}^R \right) + \\
& \quad \frac{N_c}{32\pi^2} \left( 2 (h_t^2 + h_b^2) f_{\phi}(0,0,+) \otimes P_{\phi\phi}^V + 2 (h_t^2 + h_b^2) f_{L_3}(0,0,+) \otimes P_{L\phi}^R + 2 \sum_R h_{R_3}^2 f_{R_3}(0,0,+) \otimes P_{R\phi}^R \right);
\end{aligned} \right. \quad (14)$$



$$\left\{ \begin{aligned}
& -\frac{\partial}{\partial t} f(0,0;-) = \frac{\alpha_Y}{2\pi} \left( y_R^2 f(0,0;-) \otimes (P_{ff}^V + P_{ff}^R) \right) + \\
& \quad \frac{\delta_{i3}}{32\pi^2} h_R^2 \left( 4 f(0,0;-) \otimes \mathcal{P}_{RR}^V + 4 f(0,0;-) \otimes P_{LR}^R \pm 4 f(0,0;-) \otimes P_{\phi R}^R \right) \quad (+ \text{ for up and - for down type fermions}); \\
& -\frac{\partial}{\partial t} f(0,0;-) = \left( \frac{3}{4} \frac{\alpha_W}{2\pi} + y_L^2 \frac{\alpha_Y}{2\pi} \right) f(0,0;-) \otimes (P_{ff}^V + P_{ff}^R) + \\
& \quad \frac{\delta_{i3}}{32\pi^2} \left( 2(h_t^2 + h_b^2) f(0,0;-) \otimes \mathcal{P}_{LL}^V + 2 \sum_R h_R^2 f(0,0;-) \otimes P_{RL}^R - 2(h_t^2 - h_b^2) f(0,0;-) \otimes P_{\phi L}^R \right); \\
& -\frac{\partial}{\partial t} f(00;-) = \frac{\alpha_W}{2\pi} \left( \frac{3}{4} f(00;-) \otimes (P_{\phi\phi}^V + P_{\phi\phi}^R) \right) + \frac{\alpha_Y}{2\pi} \left( \frac{1}{4} f(00;-) \otimes (P_{\phi\phi}^V + P_{\phi\phi}^R) \right) + \\
& \quad \frac{N_c}{32\pi^2} \left( 2(h_t^2 + h_b^2) f(0,0;-) \otimes \mathcal{P}_{\phi\phi}^V - 2(h_b^2 f(0,0;-) - h_t^2 f(0,0;-)) \otimes P_{R\phi}^R - 2(h_t^2 - h_b^2) f(0,0;-) \otimes P_{L\phi}^R \right);
\end{aligned} \right. \quad (15)$$

$$\left\{ \begin{aligned}
& -\frac{\partial}{\partial t} f(1,0;+) = \frac{\alpha_W}{2\pi} \left( f(1,0;+) \otimes P_{ff}^V - \frac{1}{4} f(1,0;+) \otimes (P_{ff}^V + P_{ff}^R) \right) + \frac{\alpha_Y}{2\pi} \left( y_L^2 f(1,0;+) \otimes (P_{ff}^V + P_{ff}^R) \right) + \\
& \quad \frac{\sqrt{\alpha_W \alpha_Y}}{2\pi} \left( y_L f(1,0;+) \otimes P_{gf}^R \right) + \frac{\delta_{i3}}{32\pi^2} \left( 2(h_t^2 + h_b^2) f(1,0;+) \otimes \mathcal{P}_{LL}^V - 2(h_t^2 - h_b^2) f(1,0;+) \otimes P_{\phi L}^R \right); \\
& -\frac{\partial}{\partial t} f(1,0;+) = \frac{\alpha_W}{2\pi} \left( \frac{1}{2} f(1,0;+) \otimes P_{gg}^V \right) + \frac{\sqrt{\alpha_W \alpha_Y}}{2\pi} \left( 4 y_L \sum_i N_c^i f(1,0;+) \otimes P_{fg}^R + f(1,0;+) \otimes P_{\phi g}^R \right) + \frac{\alpha_Y}{2\pi} \left( \frac{1}{2} f(1,0;+) \otimes P_{BB}^V \right); \\
& -\frac{\partial}{\partial t} f(10;+) = \frac{\alpha_W}{2\pi} \left( f(10;+) \otimes P_{\phi\phi}^V - \frac{1}{4} f(10;+) \otimes (P_{\phi\phi}^V + P_{\phi\phi}^R) \right) + \frac{\alpha_Y}{2\pi} \left( \frac{1}{4} f(1,0;+) \otimes (P_{\phi\phi}^V + P_{\phi\phi}^R) \right) + \\
& \quad \frac{\sqrt{\alpha_W \alpha_Y}}{2\pi} \frac{1}{2} f(1,0;+) \otimes P_{g\phi}^R + \frac{N_c}{32\pi^2} \left( 2(h_t^2 + h_b^2) f(1,0;+) \otimes \mathcal{P}_{\phi\phi}^V + (h_b^2 - h_t^2) f(1,0;+) \otimes P_{L\phi}^R \right);
\end{aligned} \right. \quad (16)$$

$$\left\{ \begin{aligned}
& -\frac{\partial}{\partial t} f(1,0;-) = \frac{\alpha_W}{2\pi} \left( f(1,0;-) \otimes P_{ff}^V - \frac{1}{4} f(1,0;-) \otimes (P_{ff}^V + P_{ff}^R) + \frac{1}{2} f(1,0;-) \otimes P_{gf}^R \right) + \\
& \quad \frac{\alpha_Y}{2\pi} \left( y_L^2 f(1,0;-) \otimes (P_{ff}^V + P_{ff}^R) \right) + \frac{\delta_{i3}}{32\pi^2} (h_t^2 + h_b^2) \left( 2 f(1,0;-) \otimes \mathcal{P}_{LL}^V + 2 f(1,0;-) \otimes P_{\phi L}^R \right); \\
& -\frac{\partial}{\partial t} f(1,0;-) = \frac{\alpha_W}{2\pi} \left( f(1,0;-) \otimes P_{gg}^V + f(1,0;-) \otimes (P_{gg}^V + P_{gg}^R) + \sum_i N_c^i f(1,0;-) \otimes P_{fg}^R + f(1,0;-) \otimes P_{\phi g}^R \right); \\
& -\frac{\partial}{\partial t} f(10;-) = \frac{\alpha_W}{2\pi} \left( f(10;-) \otimes P_{\phi\phi}^V - \frac{1}{4} f(1,0;-) \otimes (P_{\phi\phi}^V + P_{\phi\phi}^R) + \frac{1}{2} f(1,0;-) \otimes P_{g\phi}^R \right) + \\
& \quad \frac{\alpha_Y}{2\pi} \left( \frac{1}{4} f(10;-) \otimes (P_{\phi\phi}^V + P_{\phi\phi}^R) \right) + \frac{N_c}{32\pi^2} \left( 2(h_t^2 + h_b^2) f(1,0;-) \otimes \mathcal{P}_{\phi\phi}^V + 2(h_t^2 + h_b^2) f(1,0;-) \otimes P_{L\phi}^R \right);
\end{aligned} \right. \quad (17)$$

$$-\frac{\partial}{\partial t} f(2,0;+) = \frac{\alpha_W}{2\pi} \left( 3 f(2,0;+) \otimes P_{gg}^V - f(2,0;+) \otimes (P_{gg}^V + P_{gg}^R) \right); \quad (18)$$

$$-\frac{\partial}{\partial t} f(11;-) = \frac{\alpha_W}{2\pi} \left( f(11;-) \otimes P_{\phi\phi}^V - \frac{1}{4} f(11;-) \otimes (P_{\phi\phi}^V + P_{\phi\phi}^R) \right) + \frac{\alpha_Y}{2\pi} \left( \frac{1}{2} f(11;-) \otimes P_{\phi\phi}^V - \frac{1}{4} f(11;-) \otimes (P_{\phi\phi}^V + P_{\phi\phi}^R) \right) + \frac{N_c}{32\pi^2} \left( 2(h_t^2 + h_b^2) f(1,1;-) \otimes \mathcal{P}_{\phi\phi}^V \right); \quad (19)$$

$$-\frac{\partial}{\partial t} f(11;+) = \frac{\alpha_W}{2\pi} \left( f(11;+) \otimes P_{\phi\phi}^V - \frac{1}{4} f(11;+) \otimes (P_{\phi\phi}^V + P_{\phi\phi}^R) \right) + \frac{\alpha_Y}{2\pi} \left( \frac{1}{2} f(11;+) \otimes P_{\phi\phi}^V - \frac{1}{4} f(11;+) \otimes (P_{\phi\phi}^V + P_{\phi\phi}^R) \right) + \frac{N_c}{32\pi^2} \left( 2(h_t^2 + h_b^2) f(1,1;+) \otimes \mathcal{P}_{\phi\phi}^V \right); \quad (20)$$

$$\begin{cases}
-\frac{\partial}{\partial t} f_{u, LR_i}(\frac{1}{2}, \frac{1}{4}; \pm) = \frac{\alpha_W}{2\pi} \left( \frac{3}{8} f_{u, LR_i}(\frac{1}{2}, \frac{1}{4}; \pm) \otimes P_{ff}^V \right) + \frac{\alpha_Y}{2\pi} \left( \frac{1}{2} (y_L - y_R^u)^2 f_{u, LR_i}(\frac{1}{2}, \frac{1}{4}; \pm) \otimes P_{ff}^V + y_L y_R^u f_{u, LR_i}(\frac{1}{2}, \frac{1}{4}; \pm) \otimes (P_{ff}^V + P_{ff}^R) \right) + \\
\frac{\delta_{i3}}{32\pi^2} \left( 2 h_t^2 f_{u, LR_3}(\frac{1}{2}, \frac{1}{4}; \pm) \otimes \mathcal{P}_{RR}^V + (h_t^2 + h_b^2) f_{u, LR_3}(\frac{1}{2}, \frac{1}{4}; \pm) \otimes \mathcal{P}_{LL}^V - 2 h_t h_b f_{u, LR_3}(\frac{1}{2}, \frac{1}{4}; \pm) \otimes \mathcal{P}_{LR}^R \right); \\
-\frac{\partial}{\partial t} f_{d, LR_i}(\frac{1}{2}, \frac{1}{4}; \pm) = \frac{\alpha_W}{2\pi} \left( \frac{3}{8} f_{d, LR_i}(\frac{1}{2}, \frac{1}{4}; \pm) \otimes P_{ff}^V \right) + \frac{\alpha_Y}{2\pi} \left( \frac{1}{2} (y_L - y_R^d)^2 f_{d, LR_i}(\frac{1}{2}, \frac{1}{4}; \pm) \otimes P_{ff}^V + y_L y_R^d f_{d, LR_i}(\frac{1}{2}, \frac{1}{4}; \pm) \otimes (P_{ff}^V + P_{ff}^R) \right) + \\
\frac{\delta_{i3}}{32\pi^2} \left( 2 h_b^2 f_{d, LR_3}(\frac{1}{2}, \frac{1}{4}; \pm) \otimes \mathcal{P}_{RR}^V + (h_t^2 + h_b^2) f_{d, LR_3}(\frac{1}{2}, \frac{1}{4}; \pm) \otimes \mathcal{P}_{LL}^V - 2 h_t h_b f_{u, LR_3}(\frac{1}{2}, \frac{1}{4}; \pm) \otimes \mathcal{P}_{LR}^R \right)
\end{cases} \quad (21)$$

By looking at the above equations we can see that only structure functions that share the same set of quantum numbers are mixed by the evolution equations.

Perturbative initial conditions are independent of the particular process considered; in fact they are set at the hard scale  $\sqrt{s}$ , where no soft radiation is emitted, corresponding to tree level values for  $\mathcal{F}$ .

We have therefore (see fig.1):

$$\mathcal{F}_{\alpha'\beta'}^{\alpha\beta}(x, \mu = \sqrt{s}) = \delta_{ij} \delta_{\alpha\alpha'} \delta_{\beta\beta'} \delta(1-x) \quad (22)$$

or, expressed in rotated structure functions with total isospin and hypercharge

$$f_{(T,Y;CP)}^j(x, \mu = \sqrt{s}) = \delta_{ij} \delta(1-x) \quad (23)$$

as can be checked by their definition in eqs.(44-60).

Eqs. (13-20), together with the initial conditions (22-23) are the main results of this paper. They are the electroweak analogous of DGLAP equations for QCD, and allow to calculate the distribution of a particle  $i$  inside a particle  $j$  at a given scale  $\mu^2 \geq M_W^2$ . Only electroweak effects are treated here; a complete treatment needs to take into account also the low energy ( $\mu^2 \leq M_W^2$ ) QED running and the QCD effects of course.

As stated above, we choose to work in a gauge basis since in the high energy limit mass insertions (but not the top Yukawa insertions one) can be neglected, and calculations are simpler than in the mass eigenstates basis. However, initial (asymptotic) states are mass eigenstates and we need to rotate to this base in order to obtain physical cross sections. This can be done as a final rotation to the mass basis, once the evolved overlap matrix  $\hat{\mathcal{O}}_{ij}^i(p_1, p_2; k_1, k_2)$  in the gauge basis is obtained. We write here an example in order to clarify this procedure; an analysis at the leading double log level has already been done [15, 16, 14, 20]. We discuss the change of basis for the structure functions  $\mathcal{F}$ , but things work in exactly the same way for the overlap matrix.

Let us consider the case of an initial photon beam ( $|\gamma\rangle = c_W |\mathcal{B}\rangle + s_W |W^3\rangle$ ). The structure function in the mass eigenstate basis can be written in terms of structure functions of the corresponding gauge eigenstates:

$$\mathcal{F}_{\gamma\gamma} = s_W^2 \mathcal{F}_{g33} + 2s_W c_W \mathcal{F}_{\mathcal{B}3} + c_W^2 \mathcal{F}_{\mathcal{B}} = s_W^2 f_{g(0,0;+)} + c_W^2 f_{\mathcal{B}(0,0;+)} + 2s_W c_W f_{\mathcal{B}3(1,0;+)} - \frac{4}{3} s_W^2 f_{g(2,0;+)} \quad (24)$$

obtained expanding the direct product  $(c_W |\mathcal{B}\rangle + s_W |W^3\rangle) \otimes (c_W |\mathcal{B}\rangle + s_W |W^3\rangle)$ , and where we used (44-60) to write the  $\mathcal{F}$  in terms of  $f_{(T,Y;CP)}$ .

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## 4 Appendix:

### Kernels of the evolution equations

$$P_{ff}^V(z, k_\perp) = -\delta(1-z) \left( \log \frac{s}{k_\perp^2} - \frac{3}{2} \right); \quad P_{ff}^R(z) = \frac{1+z^2}{1-z}; \quad (25)$$

$$P_{\phi\phi}^V(z, k_\perp) = -\delta(1-z) \left( \log \frac{s}{k_\perp^2} - 2 \right); \quad P_{\phi\phi}^R(z) = 2 \frac{z}{1-z}; \quad (26)$$

$$P_{gg}^V(z, k_\perp) = -\delta(1-z) \left( \log \frac{s}{k_\perp^2} - \left( \frac{11}{6} - \frac{n_f}{6} - \frac{n_s}{24} \right) \right); \quad P_{gg}^R(z) = 2 \left( z(1-z) + \frac{z}{1-z} + \frac{1-z}{z} \right); \quad (27)$$

$$P_{\mathcal{B}\mathcal{B}}^V(z) = - \left( 2 n_f (2y_L^2 + y_{R_u}^2 + y_{R_d}^2) + y_\phi^2 \right) \delta(1-z) \frac{1}{3}$$

$$P_{gf}^R(z) = \frac{1+(1-z)^2}{z}, \quad P_{g\phi}^R(z) = 2 \frac{(1-z)}{z} \quad (28)$$

$$P_{fg}^R(z) = z^2 + (1-z)^2; \quad P_{\phi g}^R(z) = z(1-z) \quad (29)$$

$$\mathcal{P}_{LL}^V(z) = -\delta(1-z) \frac{1}{2}; \quad \mathcal{P}_{RR}^V(z) = -\delta(1-z) \frac{1}{2}; \quad \mathcal{P}_{\phi\phi}^V(z) = -\delta(1-z) \quad (30)$$

$$P_{LR}^R(z) = P_{RL}^R(z) = (1-z); \quad P_{L\phi}^R(z) = P_{R\phi}^R(z) = 1; \quad P_{\phi L}^R(z) = P_{\phi R}^R(z) = z$$

where  $n_f = \sum_i N_c^i$  is the sum over all the fermion families ( $N_c^i = 1$  for leptons and  $N_c^i = 3$  for quarks),  $n_s = 1$  is the number of Higgs doublets and  $N_c = 3$  is a color factor.

### Fermion-Scalar Sector Yukawa Interactions

Parametrizing the scalar fields in the following way

$$\Phi = \frac{1}{\sqrt{2}}(h + i\tau_a \phi_a) = \begin{pmatrix} \phi_0 & i\phi_+ \\ i\phi_- & \phi_0^* \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} h + i\phi_3 & \phi_2 + i\phi_1 \\ -\phi_2 + i\phi_1 & h - i\phi_3 \end{pmatrix} \quad (31)$$

the interaction between the scalar sector and the Left-Right fermions can be written as

$$\bar{Q}_L \Phi \mathcal{H} Q_R + \bar{Q}_R \mathcal{H}^+ \Phi^+ Q_L = \sum_{a=1}^4 \phi^a (\bar{Q}_L \Psi^a \cdot \mathcal{H} Q_R + \bar{Q}_R \mathcal{H}^+ \cdot \Psi^{a+} \Phi^+ Q_L) \quad (32)$$

where  $\Psi^a = i\tau^a$  for  $a=1,2,3$  and  $\Psi^4 = I_{2 \times 2}$  and the third family Yukawa couplings are arranged in the following matricial form

$$\mathcal{H} = \begin{pmatrix} h_t & 0 \\ 0 & h_b \end{pmatrix} \quad (33)$$

## States classification

With the above parameterization (31), the Higgs sector of the Standard Model includes the three Goldstone modes  $\phi_1, \phi_2, \phi_3$  and the Higgs field  $h$ . It is useful to arrange these four states in the complex form of the doublet/antidoublet matrix

$$\frac{1}{\sqrt{2}} \begin{pmatrix} h + i\phi_3 & \phi_2 + i\phi_1 \\ -\phi_2 + i\phi_1 & h - i\phi_3 \end{pmatrix} = \begin{pmatrix} \bar{1} & 1 \\ 2 & 2 \end{pmatrix} \quad (34)$$

which transforms as  $\Phi \rightarrow \exp[i\alpha_L^A t^A] \Phi \exp[-i\alpha_R^B t^B]$  under the  $SU(2)_L \otimes SU(2)_R$  group. The indices  $1, 2, \bar{1}, \bar{2}$  here refer to the transformation properties under  $SU(2)_L$ .

$t^A$  and  $T^A$ ,  $A = 1, 2, 3$  are the  $SU(2)_L$  generators in the fundamental and adjoint representation.

The other matrices appearing in the text are the generators of the  $SU(2)_L \otimes SU(2)_R$  group in the  $(\phi_1, \phi_2, \phi_3, h)$  basis; their explicit form is as follows:

$$\mathcal{T}_L^A = \frac{1}{2} \begin{pmatrix} i\epsilon_{BAC} & -i\delta_{AB} \\ i\delta_{AC} & 0 \end{pmatrix} \quad \mathcal{T}_R^A = \frac{1}{2} \begin{pmatrix} i\epsilon_{BAC} & i\delta_{AB} \\ -i\delta_{AC} & 0 \end{pmatrix} \quad (35)$$

Furthermore,  $(\mathcal{T}_h^3)_{ab} = i(\delta_{4a}\delta_{3b} - \delta_{3a}\delta_{4b})$  and  $(\mathcal{T}_y^3)_{ab} = \delta_{3a}\delta_{4b} + \delta_{3b}\delta_{4a}$ .

States in the Higgs sector can be classified according to their  $SU(2)_L \otimes SU(2)_R$  properties, with definite  $|\mathbf{T}, T_L^3; \mathbf{T}_R, T_R^3\rangle$  quantum numbers (see also appendix in [15]). However not all of the 16 possible states appear in the evolution equations. In fact, a physical cross section is always an overlap matrix element with a given particle in the left leg and its antiparticle in the right one. By charge conservation then, all the states involved in the evolution equations have t-channel charge equal to zero:  $Q = T_L^3 + Y = T_L^3 + T_R^3 = 0$ . Then, only 6 states are selected:

$$\begin{aligned} |1, -1; 1, 1\rangle &= -|22\rangle & |1, 1; 1, -1\rangle &= -|\bar{1}\bar{1}\rangle & |1, 0; 1, 0\rangle &= -\frac{|1\bar{2}\rangle + |\bar{1}2\rangle + |2\bar{1}\rangle + |\bar{2}1\rangle}{2} \\ |1, 0; 0, 0\rangle &= -\frac{|1\bar{2}\rangle - |\bar{1}2\rangle + |2\bar{1}\rangle - |\bar{2}1\rangle}{2} & |0, 0; 1, 0\rangle &= -\frac{|1\bar{2}\rangle + |\bar{1}2\rangle - |2\bar{1}\rangle - |\bar{2}1\rangle}{2} & |0, 0; 0, 0\rangle &= -\frac{|1\bar{2}\rangle - |\bar{1}2\rangle - |2\bar{1}\rangle + |\bar{2}1\rangle}{2} \end{aligned} \quad (36)$$

In the high energy limit we are considering the  $SU(2)_L \otimes U(1)$  symmetry is recovered, and states evolve according to their total t-channel isospin  $\mathbf{T}$  and hypercharge  $Y$ . However quantum numbers  $|\mathbf{T}, Y\rangle$  do not provide a complete classification of the states. For instance, we have two states corresponding to  $|\mathbf{T} = 0, Y = 0\rangle$ : these are the states  $|0, 0; 0, 0\rangle$  and  $|0, 0; 1, 0\rangle$  appearing in (36). We choose to add another quantum number, CP, that acts as<sup>‡</sup>:

$$1 \leftrightarrow -\bar{2} \quad 2 \leftrightarrow \bar{1} \quad \Rightarrow \quad \phi_1 \leftrightarrow -\phi_1 \quad \phi_2 \leftrightarrow \phi_2 \quad \phi_3 \leftrightarrow -\phi_3 \quad \phi_4 \equiv h \leftrightarrow \phi_4 \equiv h \quad \phi_+ \leftrightarrow -\phi_- \quad (37)$$

We can now write the states with given  $|\mathbf{T}, Y^2\rangle^{(CP)}$  quantum numbers in terms of the states  $|\mathbf{T}, T_L^3; \mathbf{T}_R, T_R^3\rangle$  classified according to their  $SU(2)_L \otimes SU(2)_R$  properties and given in (36):

$$|0, 0\rangle^{(+)} = |0, 0; 0, 0\rangle \quad |1, 0\rangle^{(-)} = |1, 0; 0, 0\rangle \quad |0, 0\rangle^{(-)} = |0, 0; 1, 0\rangle \quad (38)$$

$$|1, 0\rangle^{(+)} = |1, 0; 1, 0\rangle \quad |1, 1\rangle^{(+)} = \frac{1}{2}(|1, 1; 1, -1\rangle + |1, -1; 1, 1\rangle) \quad |1, 1\rangle^{(-)} = \frac{1}{2}(|1, 1; 1, -1\rangle - |1, -1; 1, 1\rangle) \quad (39)$$

It is now easy, by using (34), to write the above states in the  $\phi_1, \phi_2, \phi_3, h$  base and to find out the corresponding  $f$  functions. For instance:

$$|0, 0\rangle^{(+)} = |0, 0; 0, 0\rangle = -\frac{|\bar{1}\bar{2}\rangle - |\bar{1}2\rangle - |2\bar{1}\rangle + |\bar{2}1\rangle}{2} = |\phi_+\phi_- \rangle + |\phi_-\phi_+ \rangle + |\phi_3\phi_3 \rangle + |\phi_h\phi_h \rangle \quad (40)$$

This combination corresponds to

$$f_{\phi}^{(0,0;+)} = \frac{\mathcal{F}_{+-} + \mathcal{F}_{-+} + \mathcal{F}_{33} + \mathcal{F}_{hh}}{4} \quad (41)$$

In the last step we have written  $f_{\phi}^{(0,0;+)}$  as a trace over the lower indices of the  $\mathcal{F}$  operator, in order to simplify the task of writing scalar evolution equations.

<sup>‡</sup>notice that the definition of CP given in [15] is different from the one used here

We use  $SU(2) \otimes U(1)$  and CP quantum numbers also to classify states in the fermions/antifermions and gauge bosons sectors. CP acts as follows:

$$\nu \leftrightarrow \nu^* \quad e \leftrightarrow e^* \quad W_\mu^+ \leftrightarrow -W_\mu^- \quad W_\mu^3 \leftrightarrow -W_\mu^3 \quad B_\mu \leftrightarrow -B_\mu \quad (42)$$

Notice that in the fermionic sector, since CP transforms a fermion in its own antifermion, the states with defined  $SU(2) \otimes U(1)$  properties, as defined in [15]:

$$f_L^{(0)} = \frac{f_e + f_\nu}{2} \quad f_{\bar{L}}^{(0)} = \frac{f_{\bar{e}} + f_{\bar{\nu}}}{2} \quad f_L^{(1)} = \frac{f_\nu - f_e}{2} \quad f_{\bar{L}}^{(1)} = \frac{f_{\bar{e}} - f_{\bar{\nu}}}{2} \quad (43)$$

do not have definite CP values since CP transforms, for instance,  $f_L^{(0)}$  into  $f_{\bar{L}}^{(0)}$ .

SU(2) symmetry acts on the gauge bosons sector as a triplet, and classifying the states does not present any particular difficulty. We can then proceed and write all the states in the fermionic, scalar, gauge bosons sectors according to their  $|\mathbf{T}, Y^2\rangle^{CP}$  quantum numbers. Here we write directly the corresponding structure functions  $f^{(\mathbf{T}, \mathbf{Y}; CP)}$  and for Left fermions we write as example the case with  $L_i$  equal to left leptons of the first  $i = 1$  family ( $e$  and  $\nu$ ):

- singlet states  $\mathbf{T} = 0, \mathbf{Y} = 0$

$$f_{\mathbf{B}}^{(0,0;+)} \equiv \mathcal{F}_{\mathbf{B}} \quad (44)$$

$$f_{R_i}^{(0,0;+)} \equiv \frac{1}{2}(\mathcal{F}_{R_i} + \mathcal{F}_{\bar{R}_i}) \quad (45)$$

$$f_{L_i}^{(0,0;+)} \equiv \frac{1}{4} \text{Tr} \left[ \mathcal{F}_{L_i} + \mathcal{F}_{\bar{L}_i} \right] = \frac{1}{4} \left( \mathcal{F}_{L_{ee}} + \mathcal{F}_{L_{\nu\nu}} + \mathcal{F}_{L_{\bar{e}\bar{e}}} + \mathcal{F}_{L_{\bar{\nu}\bar{\nu}}} \right) \quad (46)$$

$$f_{\phi}^{(0,0;+)} \equiv \frac{1}{4} \text{Tr}[\mathcal{F}_{\phi}] = \frac{\mathcal{F}_{+-} + \mathcal{F}_{-+} + \mathcal{F}_{33} + \mathcal{F}_{hh}}{4} \quad (47)$$

$$f_g^{(0,0;+)} \equiv \frac{1}{3} \text{Tr}[\mathcal{F}_g] = \frac{\mathcal{F}_{+-} + \mathcal{F}_{33} + \mathcal{F}_{-+}}{3} \quad (48)$$

$$f_{R_i}^{(0,0;-)} \equiv \frac{1}{2}(\mathcal{F}_{R_i} - \mathcal{F}_{\bar{R}_i}) \quad (49)$$

$$f_{L_i}^{(0,0;-)} \equiv \frac{1}{4} \text{Tr} \left[ \mathcal{F}_{L_i} - \mathcal{F}_{\bar{L}_i} \right] = \frac{1}{4} \left( (\mathcal{F}_{L_{ee}} + \mathcal{F}_{L_{\nu\nu}}) - (\mathcal{F}_{L_{\bar{e}\bar{e}}} + \mathcal{F}_{L_{\bar{\nu}\bar{\nu}}}) \right) \quad (50)$$

$$f_{\phi}^{(0,0;-)} \equiv -\frac{1}{2} \text{Tr}[\mathcal{T}_R^3 \mathcal{F}_{\phi}] = \frac{\mathcal{F}_{+-} - \mathcal{F}_{-+} - i(\mathcal{F}_{h3} - \mathcal{F}_{3h})}{4} \quad (51)$$

- states  $\mathbf{T} = 1, \mathbf{Y} = 0$

$$f_{B3}^{(1,0;+)} \equiv \mathcal{F}_{B3} \quad (52)$$

$$f_{\phi}^{(1,0;+)} \equiv \text{Tr}[\mathcal{T}_L^3 \mathcal{T}_R^3 \mathcal{F}_{\phi}] = \frac{\mathcal{F}_{-+} + \mathcal{F}_{+-} - \mathcal{F}_{33} - \mathcal{F}_{hh}}{4} \quad (53)$$

$$f_{L_i}^{(1,0;+)} \equiv \frac{1}{2} \text{Tr} \left[ t^3 (\mathcal{F}_{L_i} + \mathcal{F}_{\bar{L}_i}) \right] = \frac{-\mathcal{F}_{L_{ee}} + \mathcal{F}_{L_{\nu\nu}}}{4} + \frac{-\mathcal{F}_{L_{\bar{e}\bar{e}}} + \mathcal{F}_{L_{\bar{\nu}\bar{\nu}}}}{4} \quad (54)$$

$$f_{L_i}^{(1,0;-)} \equiv \frac{1}{2} \text{Tr} \left[ t^3 (\mathcal{F}_{L_i} - \mathcal{F}_{\bar{L}_i}) \right] = \frac{-\mathcal{F}_{L_{ee}} + \mathcal{F}_{L_{\nu\nu}}}{4} - \frac{-\mathcal{F}_{L_{\bar{e}\bar{e}}} + \mathcal{F}_{L_{\bar{\nu}\bar{\nu}}}}{4} \quad (55)$$

$$f_{\phi}^{(1,0;-)} \equiv -\frac{1}{2} \text{Tr}[\mathcal{T}_L^3 \mathcal{F}] = \frac{\mathcal{F}_{+-} - \mathcal{F}_{-+} + i(\mathcal{F}_{h3} - \mathcal{F}_{3h})}{4} \quad (56)$$

$$f_g^{(1,0;-)} \equiv -\frac{1}{2} \text{Tr}[\mathcal{T}_L^3 \mathcal{F}] = \frac{\mathcal{F}_{+-} - \mathcal{F}_{-+}}{2} \quad (57)$$

- states  $\mathbf{T} = 2, \mathbf{Y} = 0, CP = (+)$

$$f_g^{(2,0;+)} \equiv \frac{1}{4} \text{Tr}[(3(\mathcal{T}_L^3)^2 - 2) \mathcal{F}] = \frac{\mathcal{F}_{+-} - 2 \mathcal{F}_{33} + \mathcal{F}_{-+}}{4} \quad (58)$$

- states  $\mathbf{T} = 1, \mathbf{Y}^2 = 1, CP = (-)$

$$f_{\phi}^{(1,1;-)} \equiv \frac{1}{2} \text{Tr}[\mathcal{T}_y^3 \mathcal{F}] = \frac{\mathcal{F}_{3h} + \mathcal{F}_{h3}}{2} \quad (59)$$

- states  $\mathbf{T} = 1, \mathbf{Y}^2 = 1, CP = (+)$

$$f_{\phi}^{(1,1;+)} \equiv \frac{i}{2} \text{Tr}[(\mathcal{T}_y^3 \mathcal{T}_h^3) \mathcal{F}] = \frac{\mathcal{F}_{33} - \mathcal{F}_{hh}}{2} \quad (60)$$

- states  $\mathbf{T} = 1/2, \mathbf{Y}^2 = \frac{1}{4}$

$$f_{LR_i}^u(\frac{1}{2}, \frac{1}{4}; \pm) \equiv \frac{1}{2} \left( \mathcal{F}_{11} \pm \mathcal{F}_{\bar{L}\bar{R}_i} \right) \quad (61)$$

$$f_{LR_i}^d(\frac{1}{2}, \frac{1}{4}; \pm) \equiv \frac{1}{2} \left( \mathcal{F}_{22} \pm \mathcal{F}_{\bar{L}\bar{R}_i} \right) \quad (62)$$

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